MA / MSCMT - 02

December - Examination 2015

M.A. / M.Sc. Previous Examination

Real Analysis and Topology

Paper - MA / MSCMT - 02

Time : 3 Hours] **[Max. Marks :- 80**

Note : The question paper is divided into three sections A, B and C. Use of calculator is allowed in this paper.

Section - A 8 x 2 = 16

Note : Section 'A' contain 08 very short answer type questions. Examinees have to attempt all questions. Each question is of 02 marks and maximum word limit is 30 words.

1) (i) Define measurable sets.

(ii) What is step function?

(iii) Define co-countable topology.

(iv) Define T_2 - space.

(v) Define product topology.

(vi) Define Embedding.

(vii) Define sub-base of a filter.

(viii) Define second countable space.

Section - B 4 x 8 = 32

Note : Section 'B' contain 8 short answer type questions. Examinees will have to answer any four 04 questions. Each question is of 08 marks Examinees have to delimit each answer in maximum 200 words.

2) Prove that every open interval is a Borel set.

3) Show that outer measure is translation invariant.

4) If $< f_n >$ is a sequence of measurable functions defined on a measurable set E, then prove that $sup_n < f_n >$ and $inf_n < f_n >$ are also measurable on E.

5) Show that the space L_2 of a square summable function is a linear space.

6) Show that L^P - space is a normed metric space.

7) If A be a subset of a topological space then prove that $\overline{A} = A \cup A'$, where symbols has their usual meanings.

8) Show that a closed subset of a compact space is compact.

9) Give an example of a locally connected space which is not connected.

Section - C 2 x 16 = 32

Note : Section 'C' contain 04 Long answer type questions. Examinees will have to answer any two (02) questions. Each question is of 16 marks. Examinees have to delimit each answer in maximum 500 words.

10) Prove that the Fourier series of any function $f \in L_2$ converges in norm. Further it converges to f if and only if $||f||^2 = \sum_{i=1}^{\infty} a_1^2$ where a_i are the Fourier coefficients of the function f. If $\sum_{i=1}^{\infty} a_i f_i$ is the Fourier series of the square summable function with respect to an orthonormal sequence $< f_n >$ of square summable functions, then show that $\sum_{i=1}^{\infty} a_i^2 \leq ||f||^2$.

11) (i) Show that second countable space is always first countable but converse is not true.

(ii) Prove that every second countable regular space is normal space.

12) (i) Show that closure of a connected set in connected.

(ii) Show that one point compactification of set of rational numbers Q is not Hausdorff.

13) (i) Show that the product space (X × Y, W) is compact iff each of the spaces (X, τ) and (Y, V) is compact.

(ii) Prove that the sequence of functions in L^P space has at most one limit.